

# Microscopic Theory of Spin Transfer Torques in Ferromagnets

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Spin transfer torque (STT), involving both adiabatic and non-adiabatic contributions, provides key mechanisms to current-induced phenomena in ferromagnets. While the adiabatic STT has been well studied, current understanding of the non-adiabatic STT is based on macroscopic models. By computing microscopically the response of conduction electron spins to a time varying and spatially inhomogeneous magnetic background, we derive the adiabatic and non-adiabatic STT in a unified fashion. Our result justifies the macroscopic theory (Phys. Rev. Lett. **93**, 127204 (2004)) and clarifies the physical origin of every coefficient. We also provide a benchmark on the validity of the theory and explain why a recent experiment shows deviation.

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Interplay between current and magnetization is the central topic of spintronics [1]. When a current flows through a ferromagnetic metal, it becomes spin-polarized due to local exchange coupling between conduction electron spins and magnetic moments. In turn, spin angular momentum is transferred from current to magnetization regarding the constraint of spin conservation, which is now known as the spin transfer torque (STT) [2, 3]. STT provides key mechanisms to numerous intriguing phenomena in ferromagnets, for example, current-driven domain wall motion [4, 5], spin wave excitations [6, 7], *etc.* In both fundamental physics and application oriented studies, STT has aroused enormous attention in the last two decades [8, 9]. However, while magnetization dynamics driven by STT is becoming the core issue of spintronics, fundamental physics underlying STT is far from clear.

At present, STT is believed to be divided into adiabatic (reactive) and non-adiabatic (dissipative) contributions. While the former has been derived microscopically via different approaches [10, 11], the latter has only been justified macroscopically through spin conservation and Galilean invariance [3, 12–14], whose microscopic origin is under intense debates. In many recent efforts, microscopic theories have been developed in generic ways [15–18] and in specific contexts [19–21], but their coefficients do not reach a consensus, meanwhile, some others even cast doubt on the existence of the non-adiabatic STT [22]. On the other hand, experimental measurements of this torque also exhibits various results that are not in agreement [23–25].

In this Letter, a microscopic derivation of the magnetization dynamics induced by STT is provided. We first calculate the response of a conduction electron spin to a time varying and spatially inhomogeneous magnetic background  $\mathbf{M}(\mathbf{r}, t)$ , then obtain the non-equilibrium local spin accumulation  $\delta\mathbf{m}$  (perpendicular to  $\mathbf{M}(\mathbf{r}, t)$ ) by integration over the Brillouin Zone. Due to the exchange coupling between itinerant electron spins and background magnetization, the back action of current exerting on

$\mathbf{M}(\mathbf{r}, t)$  is proportional to  $\delta\mathbf{m} \times \mathbf{M}$ , where the adiabatic and non-adiabatic STT naturally appear on an equal footing. Our result justifies the macroscopic model proposed in Ref. [3] with all coefficients matched exactly. At the end of this Letter, a benchmark on the validity of our theory is given and we explain why a recent measurement on narrow domain walls [25] shows deviations from theoretical prediction.

To avoid confusion of symbols, we adopt the convention used in Ref. [3]. The exchange interaction between itinerant electrons and local magnetic moments is governed by the Hamiltonian,

$$H_{ex} = \frac{SJ_{ex}}{M_s} \mathbf{s} \cdot \mathbf{M}(\mathbf{r}, t) \quad (1)$$

where  $\mathbf{s}$  is the (dimensionless) spin of a conduction electron,  $|\mathbf{M}(\mathbf{r}, t)| = M_s$  is the saturation magnetization, and  $S$  denotes the magnitude of background spins. The coupling strength  $J_{ex}$  can be as large as eV in typical transition metals, so that if  $\mathbf{M}(\mathbf{r}, t)$  varies slowly in space and time, conduction electron spins will follow the background profile when the system is in thermal equilibrium, which is known as the adiabatic limit. However, when an external current is applied to the system, a small non-equilibrium spin accumulation  $\delta\mathbf{m}$  transverse to local  $\mathbf{M}(\mathbf{r}, t)$  is induced. It is this  $\delta\mathbf{m}$  that exerts STT to the background magnetization.

To compute  $\delta\mathbf{m}$ , we first study the spin response of an individual conduction electron to the background  $\mathbf{M}(\mathbf{r}, t)$  when current is applied. From Eq. (1), we know that *local* spin-up (majority) and spin-down (minority) bands are separated by a large gap  $\Delta \equiv SJ_{ex} = \mathcal{E}_\downarrow - \mathcal{E}_\uparrow$ , and the associated spin wave functions are denoted by  $|\uparrow(\mathbf{r}, t)\rangle$  and  $|\downarrow(\mathbf{r}, t)\rangle$ . The electron is described by a coherent wave packet centered at  $(\mathbf{r}_c, \mathbf{k}_c)$  [26, 27],

$$|W\rangle = \int d^3\mathbf{k} w(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}} |\mathbf{k}\rangle [c_a |\uparrow(\mathbf{r}_c, t)\rangle + c_b |\downarrow(\mathbf{r}_c, t)\rangle] \quad (2)$$

where  $w(\mathbf{k})$  satisfying  $\int d\mathbf{k} \mathbf{k} |w(\mathbf{k})|^2 = \mathbf{k}_c$  is the profile function,  $|\mathbf{k}\rangle$  is the periodic part of *local* Bloch function,

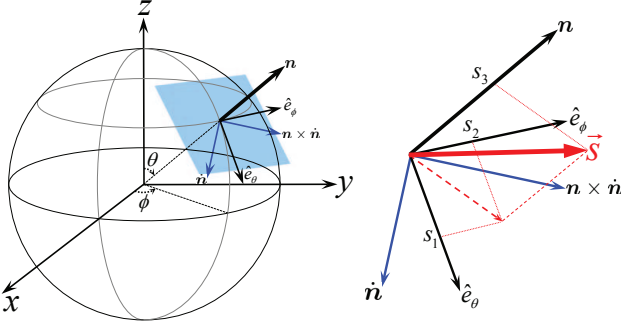


FIG. 1: (Color online) Eigenstates of Hamiltonian Eq. (1) form a set of local spin basis, and defines a local frame that moves with  $\mathbf{n} = \mathbf{M}/M_s$ . Components of the conduction electron spin  $\mathbf{s}$  (red) in the local frame are denoted by  $s_1$ ,  $s_2$ , and  $s_3$ . In the tangential plane with normal  $\mathbf{n}$ , we make a coordinate transformation from  $\hat{e}_\theta$  and  $\hat{e}_\phi$  to  $\hat{\mathbf{n}}$  and  $\mathbf{n} \times \hat{\mathbf{n}}$  so that everything expressed in the new basis is physical.

and  $c_a$ ,  $c_b$  are superposition coefficients. Now  $|\uparrow(\mathbf{r}_c, t)\rangle$  and  $|\downarrow(\mathbf{r}_c, t)\rangle$  form a set of local spin basis with quantization axis being  $\mathbf{n}(\mathbf{r}_c, t) = \mathbf{M}(\mathbf{r}_c, t)/M_s$ , therefore we can construct a local frame moving with  $\mathbf{M}(\mathbf{r}_c, t)$  where the coordinates are labeled by  $\mathbf{n}$ ,  $\hat{e}_\theta$ , and  $\hat{e}_\phi$  in Fig. 1. The electron spin expressed in this local frame reads,

$$\begin{aligned} \mathbf{s} &= \{s_1, s_2, s_3\} = \eta^\dagger \boldsymbol{\sigma} \eta \\ &= \{2\text{Re}(c_a c_b^*), -2\text{Im}(c_a c_b^*), |c_a|^2 - |c_b|^2\}, \end{aligned} \quad (3)$$

where  $\boldsymbol{\sigma}$  is a vector of Pauli matrices, and  $\eta = [c_a, c_b]^T$  is regarded as spin wave function in the local basis.

The equations of motion are obtained from the universal Lagrangian  $L = \langle W | i\hbar \partial_t - H | W \rangle$  through variational principle [26], which involves not only dynamics of  $\mathbf{r}_c$  and  $\mathbf{k}_c$ , but dynamics between the two (well separated) spin bands as well. The latter represents spin evolution with respect to the local magnetization  $\mathbf{M}(\mathbf{r}, t)$  and exhibits fast rotating character due to the large gap  $\Delta$ , it should be distinguished with the adiabatic dynamics between degenerate bands [27]. Because of space-time dependence of the local spin basis, Berry gauge connections are induced in the effective Lagrangian [28],

$$L = i\hbar \eta^\dagger \dot{\eta} + \eta^\dagger [\dot{\mathbf{r}}_c \cdot \mathbf{A} + \Phi] \eta + \hbar \mathbf{k}_c \cdot \dot{\mathbf{r}}_c - \frac{1}{2} s_3 \Delta - \mathcal{E}_0, \quad (4)$$

where  $\mathcal{E}_0 = \frac{1}{2}(\mathcal{E}_\uparrow(\mathbf{k}_c) + \mathcal{E}_\downarrow(\mathbf{k}_c))$  denotes the average of local band energy; the gap  $\Delta$  couples only  $s_3$ , which resembles a local Zeeman energy. The Berry connections have both spatial component

$$\mathbf{A} = i\hbar \begin{bmatrix} \langle \uparrow | \nabla | \uparrow \rangle & \langle \uparrow | \nabla | \downarrow \rangle \\ \langle \downarrow | \nabla | \uparrow \rangle & \langle \downarrow | \nabla | \downarrow \rangle \end{bmatrix} \quad (5)$$

as a vector potential ( $\nabla = \frac{\partial}{\partial \mathbf{r}_c}$ ), and temporal component

$$\Phi = i\hbar \begin{bmatrix} \langle \uparrow | \partial_t | \uparrow \rangle & \langle \uparrow | \partial_t | \downarrow \rangle \\ \langle \downarrow | \partial_t | \uparrow \rangle & \langle \downarrow | \partial_t | \downarrow \rangle \end{bmatrix} \quad (6)$$

as a scalar potential. The local spin wave functions are taken to be  $|\uparrow\rangle = [e^{-i\frac{\phi}{2}} \cos \frac{\theta}{2}, e^{i\frac{\phi}{2}} \sin \frac{\theta}{2}]^T$  and  $|\downarrow\rangle = [-e^{-i\frac{\phi}{2}} \sin \frac{\theta}{2}, e^{i\frac{\phi}{2}} \cos \frac{\theta}{2}]^T$ , where  $\theta = \theta(\mathbf{r}_c, t)$  and  $\phi = \phi(\mathbf{r}_c, t)$  are spherical angles specifying the direction of  $\mathbf{M}(\mathbf{r}_c, t)$ , whose total time derivatives are  $\dot{\theta} = \dot{\mathbf{r}}_c \cdot \nabla \theta + \partial_t \theta$  and  $\dot{\phi} = \dot{\mathbf{r}}_c \cdot \nabla \phi + \partial_t \phi$ . Then the Berry connection terms can be written as a unified  $2 \times 2$  matrix,

$$\dot{\mathbf{r}}_c \cdot \mathbf{A} + \Phi = \frac{\hbar}{2} \begin{bmatrix} \cos \theta \dot{\phi} & -\sin \theta \dot{\phi} - i\dot{\theta} \\ -\sin \theta \dot{\phi} + i\dot{\theta} & -\cos \theta \dot{\phi} \end{bmatrix}. \quad (7)$$

It is worth mentioning that freedom exists in the choice of local spin wave functions, which leads to the gauge freedom of the Berry gauge connections. More graphically, a specified set of spin wave functions corresponds to a particular choice of local frame in Fig. 1, thus the relative orientation of the local frame can be rotated about  $\mathbf{n}$  by gauge transformations, and is not physical. But everything will be expressed in terms of gauge invariant quantities at the end.

Regarding Eq. (3), the spin dynamics is obtained through  $\delta L / \delta \eta = 0$  after some manipulations [28],

$$\begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \\ \dot{s}_3 \end{bmatrix} = \begin{bmatrix} 0 & \cos \theta \dot{\phi} - \frac{1}{\tau_{ex}} & -\dot{\theta} \\ -\cos \theta \dot{\phi} + \frac{1}{\tau_{ex}} & 0 & -\sin \theta \dot{\phi} \\ \dot{\theta} & \sin \theta \dot{\phi} & 0 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} \quad (8)$$

where  $\tau_{ex} = \Delta/\hbar$  is defined as the exchange time. Eq. (8) describes coherent spin dynamics in the local frame moving with  $\mathbf{M}(\mathbf{r}_c, t)$ . However, spin relaxation as a non-coherent process should also be taken into account. In real materials spin relaxation is very case dependent, but regardless of the underlying mechanism, it adds a term  $-\frac{1}{\tau_{sf}}(\mathbf{s} - \mathbf{s}_{eq})$  to Eq. (8), where  $\tau_{sf}$  is the mean spin-flip time and  $\mathbf{s}_{eq} = \{0, 0, 1(-1)\}$  is the local equilibrium spin configuration for the majority (minority) band  $\mathcal{E}_\uparrow(\mathcal{E}_\downarrow)$ . Eq. (8) should be solved numerically in general, but approximation can be made upon the following considerations: the large gap  $\Delta$  results in an extremely small  $\tau_{ex}$  (typically of the order of  $10^{-14} \sim 10^{-15} \text{s}$ ), thus within several  $\tau_{ex}$ s the change of magnetization seen by the electron is small, *i.e.*,  $\partial_t \mathbf{M}$  and  $(\dot{\mathbf{r}}_c \cdot \nabla) \mathbf{M}$  is small compared to  $1/\tau_{ex}$ . To this end, we define two small parameters  $\varepsilon_1 = \tau_{ex} \sin \theta \dot{\phi}$  and  $\varepsilon_2 = \tau_{ex} \dot{\theta}$  which satisfy  $\sqrt{\varepsilon_1^2 + \varepsilon_2^2} = \hbar |\dot{\mathbf{M}}| / (M_s \Delta) \ll 1$ . On the same time scale, the variations of  $\varepsilon_1$  and  $\varepsilon_2$  are even higher order small quantities, thus it is a good approximation to treat  $\varepsilon_1$  and  $\varepsilon_2$  as constants, by which Eq. (8) becomes a set of first order differential equations with constant coefficient matrix. As a result, it can be solved analytically. As explained in the supplementary material [28], given the initial condition  $\mathbf{s} = \mathbf{s}_{eq}$ , the solution of Eq. (8) for the majority band reads,

$$s_1(t) = \frac{\varepsilon_1 - \xi \varepsilon_2}{1 + \xi^2} - \frac{e^{-\xi \tilde{t}}}{1 + \xi^2} [\varepsilon_1(\cos \tilde{t} + \xi \sin \tilde{t}) + \varepsilon_2(\sin \tilde{t} - \xi \cos \tilde{t})] \quad (9a)$$

$$s_2(t) = -\frac{\xi \varepsilon_1 + \varepsilon_2}{1 + \xi^2} - \frac{e^{-\xi \tilde{t}}}{1 + \xi^2} [\varepsilon_1(\sin \tilde{t} - \xi \cos \tilde{t}) - \varepsilon_2(\cos \tilde{t} + \xi \sin \tilde{t})] \quad (9b)$$

$$s_3(t) = 1 + \frac{e^{-\xi \tilde{t}}}{1 + \xi^2} (\varepsilon_1^2 + \varepsilon_2^2) [\cos \tilde{t} + \xi \sin \tilde{t}] \quad (9c)$$

where  $\tilde{t} = t/\tau_{ex}$  is the scaled time and  $\xi = \tau_{ex}/\tau_{sf}$ . All the time-dependent terms in Eq. (9) exhibit oscillatory pattern along with the exponentially decaying factor  $e^{-\xi \tilde{t}}$ . Since  $\tau_{ex}$  is extremely small, those are fast rotating terms (also rapidly vanishing controlled by  $\xi$ ), comparing with the slow dynamics of  $\mathbf{M}$ . Therefore in deriving the dynamics of  $\mathbf{M}$  those time dependencies are averaged out, and we have

$$\langle s_1 \rangle = \frac{\varepsilon_1 - \xi \varepsilon_2}{1 + \xi^2}, \quad (10a)$$

$$\langle s_2 \rangle = -\frac{\xi \varepsilon_1 + \varepsilon_2}{1 + \xi^2}, \quad (10b)$$

$$\langle s_3 \rangle = 1, \quad (10c)$$

and the spin of the electron is  $\mathbf{s} = \mathbf{s}_{eq} + \delta \mathbf{s}$  where  $\delta \mathbf{s} = \langle s_1 \rangle \hat{e}_\theta + \langle s_2 \rangle \hat{e}_\phi$ . For the minority band, Eq. (10) only differs by an overall minus sign. To express  $\delta \mathbf{s}$  in terms of gauge invariant quantities, we need to make a coordinate transformation which corresponds to a rotation of basis in the tangential plane depicted in Fig. 1,

$$\begin{bmatrix} \dot{\mathbf{n}} \\ \mathbf{n} \times \dot{\mathbf{n}} \end{bmatrix} = \frac{\Omega}{\tau_{ex}} \begin{bmatrix} \varepsilon_2 & \varepsilon_1 \\ -\varepsilon_1 & \varepsilon_2 \end{bmatrix} \begin{bmatrix} \hat{e}_\theta \\ \hat{e}_\phi \end{bmatrix} \quad (11)$$

where  $\Omega = |\dot{\mathbf{n}}|$ . Then we obtain,

$$\begin{aligned} \delta \mathbf{s}_{\uparrow, \downarrow} &= \mp \frac{\tau_{ex}}{1 + \xi^2} [\mathbf{n} \times \dot{\mathbf{n}} + \xi \dot{\mathbf{n}}] \\ &= \mp \frac{\tau_{ex}}{1 + \xi^2} [\mathbf{n} \times \frac{\partial \mathbf{n}}{\partial t} + \xi \frac{\partial \mathbf{n}}{\partial t} \\ &\quad + \mathbf{n} \times (\dot{\mathbf{r}}_c \cdot \nabla) \mathbf{n} + \xi (\dot{\mathbf{r}}_c \cdot \nabla) \mathbf{n}], \end{aligned} \quad (12)$$

where  $\dot{\mathbf{n}} = \partial_t \mathbf{n} + (\dot{\mathbf{r}}_c \cdot \nabla) \mathbf{n}$  has been used and  $\dot{\mathbf{r}}_c = -\frac{\partial \mathcal{E}_{\uparrow, \downarrow}}{\hbar \partial \mathbf{k}_c}$  is the center of mass velocity. The local non-equilibrium spin accumulation is obtained by integration

$$\delta \mathbf{m} = \mu_B \int d\mathcal{E} [\mathcal{D}_\uparrow(\mathcal{E}) g_\uparrow(\mathcal{E}) \delta \mathbf{s}_\uparrow + \mathcal{D}_\downarrow(\mathcal{E}) g_\downarrow(\mathcal{E}) \delta \mathbf{s}_\downarrow] \quad (13)$$

where  $\mu_B$  is the Bohr magneton,  $\mathcal{D}_{\uparrow, \downarrow}(\mathcal{E})$  is the density of states, and  $g_{\uparrow, \downarrow}(\mathcal{E})$  represents the distribution function. In weak electric field  $\mathbf{E}$  and zero temperature, we have  $g_{\uparrow, \downarrow}(\mathcal{E}) = f_{0\uparrow, \downarrow}(\mathcal{E}) + e\tau_{0\uparrow, \downarrow} \mathbf{E} \cdot \frac{\partial \mathcal{E}_{\uparrow, \downarrow}}{\hbar \partial \mathbf{k}_c} \frac{\partial f_{0\uparrow, \downarrow}}{\partial \mathcal{E}}$  where  $f_{0\uparrow, \downarrow}(\mathcal{E})$  is the Fermi distribution function without electric field and  $\tau_{0\uparrow, \downarrow}$  is the relaxation time. Attention should be

paid that when the mean spin-flip time  $\tau_{sf}$  is assumed to be independent of energy, it is equivalent to introduce it either in solving the Boltzmann equation or in Eq. (8), and we have chosen the latter. Our target now is to relate  $\delta \mathbf{m}$  to the charge current

$$\mathbf{j}_e = -\frac{e}{\hbar} \int d\mathcal{E} \left[ \mathcal{D}_\uparrow(\mathcal{E}) g_\uparrow(\mathcal{E}) \frac{\partial \mathcal{E}_\uparrow}{\partial \mathbf{k}_c} + \mathcal{D}_\downarrow(\mathcal{E}) g_\downarrow(\mathcal{E}) \frac{\partial \mathcal{E}_\downarrow}{\partial \mathbf{k}_c} \right].$$

After some manipulations, the electric field  $\mathbf{E}$  and  $\tau_{0\uparrow, \downarrow}$  are eliminated, and we arrive at our central result,

$$\begin{aligned} \delta \mathbf{m} &= \frac{\tau_{ex}}{1 + \xi^2} \left[ -\frac{n_0}{M_s^2} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} - \frac{\xi n_0}{M_s} \frac{\partial \mathbf{M}}{\partial t} \right. \\ &\quad \left. + \frac{\mu_B P}{e M_s^2} \mathbf{M} \times (\mathbf{j}_e \cdot \nabla) \mathbf{M} + \frac{\xi \mu_B P}{e M_s} (\mathbf{j}_e \cdot \nabla) \mathbf{M} \right], \end{aligned} \quad (14)$$

where  $P = (n_\uparrow - n_\downarrow)/(n_\uparrow + n_\downarrow)$  is the spin polarization with  $n_{\uparrow(\downarrow)}$  being the electron density of the two bands at Fermi surface, and  $n_0 = \mu_B \int d\mathcal{E} \mathcal{D}(\mathcal{E}) (f_{0\uparrow}(\mathcal{E}) - f_{0\downarrow}(\mathcal{E}))$  is the local equilibrium spin density whose direction is parallel to the magnetization. Eq. (14) is exactly the same as Eq. (8) in Ref. [3], but the above derivation is purely microscopic, and the four terms of Eq. (14) can be traced back to the four terms in Eq. (12).

From Eq. (1), the STT is  $\mathbf{T} = (1/\tau_{ex} M_s) \delta \mathbf{m} \times \mathbf{M}$ , which should be added to the Landau-Lifshitz-Gilbert equation:  $\partial \mathbf{M} / \partial t = \gamma \mathbf{H}_{eff} \times \mathbf{M} + (\alpha / M_s) \mathbf{M} \times \partial \mathbf{M} / \partial t + \mathbf{T}$ , where  $\gamma$  is the gyromagnetic ratio,  $\mathbf{H}_{eff}$  is the effective magnetic field, and  $\alpha$  is the Gilbert damping parameter. The final form of magnetization dynamics becomes,

$$\begin{aligned} \frac{\partial \mathbf{M}}{\partial t} &= \tilde{\gamma} \mathbf{H}_{eff} \times \mathbf{M} + \tilde{\alpha} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} \\ &\quad + \frac{1}{1 + \eta} \left[ 1 - \xi \frac{M}{M_s} \right] \times (\mathbf{u} \cdot \nabla) \mathbf{M}, \end{aligned} \quad (15)$$

where  $\mathbf{u} = P \mathbf{j}_e \mu_B / e M_s (1 + \xi^2)$  is the effective electron velocity, and  $\eta = (n_0 / M_s) / (1 + \xi^2)$  is a dimensionless factor. The renormalized gyromagnetic ratio and Gilbert damping parameter are

$$\tilde{\gamma} = \frac{\gamma}{1 + \eta}, \quad \tilde{\alpha} = \frac{1}{1 + \eta} \left[ \alpha + \frac{n_0}{M_s^2} \left( \frac{\xi}{1 + \xi^2} \right) \right], \quad (16)$$

where the renormalization originates from the first two terms of Eq. (12) (or Eq. (14)), they are determined by

the local equilibrium spin density  $n_0$  which exists even in the absence of current. Eqs. (15) and (16) justify the results of previous macroscopic theory [3]

Our microscopic derivation rests on two assumptions: local equilibrium can be defined; and  $\mathbf{M}$  is nearly constant within several  $\tau_{ex}$ s. The former requires diffusive transport which is usually the case in permalloy materials; the latter, however, is only true when the characteristic length of the texture  $l$  (*e.g.*, domain wall width) satisfies  $l \gg v_F \tau_{ex}$  where  $v_F$  is the fermi velocity, otherwise the solution Eqs. (9) and (10) are invalid. In a recent experiment [25], people measured the non-adiabatic torque on very narrow domain walls ( $1 \sim 10$ nm) and find disagreement with Eq. (15). A rough estimate using  $v_F \sim 3 \times 10^5$ m/s and  $\Delta \sim 1$ eV tells us that  $v_F \tau_{ex}$  is of the order of a few angstroms, thus a domain wall of a few nm wide cannot be considered  $l \gg v_F \tau_{ex}$ . In that case, our local solution is no longer a good approximation, and STT may exhibits non-local behavior and also oscillatory patterns as the time-dependent terms in Eq. (9) become important. Two final remarks: spin-orbital interactions may contribute an anomalous velocity to  $\dot{\mathbf{r}}_c$ , which brings additional terms into Eq. (14); the spin motive force [29]

$$\mathbf{E}_{SMF} = \frac{\hbar}{2e} \mathbf{n} \cdot (\partial_t \mathbf{n} \times \nabla \mathbf{n}),$$

though small, should be considered in a strict sense, and the electric field should be replaced by the effective field  $\mathbf{E}_{eff} = \mathbf{E} + \mathbf{E}_{SMF}$  in deriving Eq. (14) from Eqs. (12) and (13). This creates an additional contribution to the Gilbert damping parameter, which has been studied recently via quite a different route [30].

In summary, we have derived the current-induced STT including both adiabatic and non-adiabatic contributions through a microscopic point of view, which justifies the previous macroscopic theory in Ref. [3] with all coefficients matched exactly. The validity of our theory is challenged when the characteristic length of the background magnetization becomes comparable to  $v_F \tau_{ex}$ , which has been observed on narrow domain walls [25].

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- [1] I. Žutić, J. Fabian, and S. D. Sarma, Rev. Mod. Phys. **76**, 323 (2004) and the reference therein.
- [2] L. Berger, Phys. Rev. B **54**, 9353 (1996); J. Slonczewski, J. Magn. Magn. Mater. **159**, L1 (1996).
- [3] S. Zhang and Z. Li, Phys. Rev. Lett. **93**, 127204 (2004).
- [4] G. S. D. Beach, M. Tsoi, J. L. Erskine, J. Magn. Magn. Mater. **320**, 1272 (2008).
- [5] Y. Tserkovnyak, A. Brattas, and G. E. W. Bauer, J. Magn. Magn. Mater. **320**, 1282 (2008).
- [6] S. Zhang and Z. Li, Phys. Rev. Lett. **92**, 207203 (2004).
- [7] Y. Ji, C. L. Chien, and M. D. Stiles, Phys. Rev. Lett. **90**, 106601 (2003).
- [8] D. C. Ralph and M. D. Stiles, J. Magn. Magn. Mater. **320**, 1190 (2008).
- [9] A. Brataas, A. D. Kent, and H. Ohno, Nature Materials **11**, 372 (2012).
- [10] Y. B. Bazaliy, B. A. Jones, and S. -C. Zhang, Phys. Rev. B **57**, R3213 (1998).
- [11] G. Tatara and H. Kohno, Phys. Rev. Lett. **92**, 086601 (2004).
- [12] A. Thiaville, Y. Nakatani, J. Miltat, and Y. Suzuki, Europhys. Lett., **69**, 990 (2005).
- [13] S. E. Barnes and S. Maekawa, Phys. Rev. Lett. **95**, 107204 (2005).
- [14] C. H. Wong and Y. Tserkovnyak, Phys. Rev. B **80**, 184411 (2009); *ibid.* **79**, 014402 (2009).
- [15] Y. Tserkovnyak, H. J. Skadsem, A. Brataas, and G. E. W. Bauer, Phys. Rev. B **74**, 144405 (2006).
- [16] F. Piéchon and A. Thiaville, Phys. Rev. B **75**, 174414 (2007).
- [17] G. Tatara and P. Entel, Phys. Rev. B **78**, 064429 (2008); H. Kohno, G. Tatara, and J. Shibata, J. Phys. Soc. Jpn. **75**, 113706 (2006).
- [18] R. A. Duine, Phys. Rev. B **79**, 014407 (2009); R. A. Duine, A. S. Núñez, J. Sinova, and A. H. MacDonald, Phys. Rev. B **75**, 214420 (2007).
- [19] X. Waintal and M. Viret, Europhys. Lett. **65**, 427 (2004).
- [20] A. Vanhaverbeke and M. Viret, Phys. Rev. B **75**, 024411 (2007).
- [21] T. Taniguchi, J. Sato, and H. Imamura, Phys. Rev. B **79**, 212410 (2009).
- [22] J. Xiao, A. Zangwill, and M. D. Stiles, Phys. Rev. B **73**, 54428 (2006).
- [23] G. Meier *et al.*, Phys. Rev. Lett. **98**, 187202 (2007); L. Thomas *et al.*, Nature (London), **443**, 197 (2006).
- [24] L. Heyne *et al.*, Phys. Rev. Lett. **105**, 187203 (2010); L. Heyne *et al.*, Phys. Rev. Lett. **100**, 066603 (2008).
- [25] C. Burrowes *et al.*, Nature Physics **6**, 17 (2010).
- [26] D. Xiao, M. -C. Zhang, and Q. Niu, Rev. Mod. Phys. **82**, 1959 (2010) and the reference therein.
- [27] R. Cheng and Q. Niu, arXiv:1211.0782 (2012).
- [28] See the Supplementary material of this article: link.
- [29] S. A. Yang, *et al.*, Phys. Rev. Lett. **102**, 067201 (2009).
- [30] S. Zhang and S. -L. Zhang, Phys. Rev. Lett. **102**, 086601 (2009).

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